Multi-Dimensional Minimum Weighted Norm Interpolation, Survey Regularization and AVA Imaging

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TOPICS

<u>Band-limited signal interpolation</u>

- Minimum norm reconstruction
- Minimum weighed norm reconstruction
- FFT+PCG

<u>Survey Regularization</u>

- 3D DSR Depth Imaging
- 3D DSR AVA Imaging

<u>Future Directions</u>

MOTIVATION

AVA imaging: Lack of amplitude fidelity due to uneven distribution of offsets in CDP bins

Solution:

- Survey Regularization
- LS AVA Imaging
- or, some combination of the above.

TWO PROCESSING SCHEMES

Include sampling considerations at the time of inverting an Earth model (LS Migration)

- LS AVA Migration, H. Kuehl, UofA PhD Thesis, 2002
- 3D AVA Wave equation migration, J. Wang, PhD in progress

Interpolate then invert/migrate

- Pre-stack data regularization, B. Liu, PhD in progress

Survey Regularization

Minimum weigthed Norm Interpolation

- Interpolate
- Migrate

Band-Limited Interpolation
1D Problem

Complete data: $\mathbf{x} = [x_1, x_2, x_3, \ldots x_M]^T$

Observations: $\mathbf{y} = [x_{n(1)}, x_{n(2)}, x_{n(3)}, \ldots x_{n(N)}]^T$

Sampling set: $\mathcal{N} = \{n(1), n(2), n(3), \ldots, n(N)\}$

Sampling Operator: $\mathbf{y} = \underset{\sim}{\mathbf{T}}\mathbf{x}$

$T_{i,j} = \delta_{n(i),j}$

$\underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{T}}^T = \underset{\sim}{\mathbf{I}}_N \,, \quad \underset{\sim}{\mathbf{T}}^T\underset{\sim}{\mathbf{T}} \neq \underset{\sim}{\mathbf{I}}_M$

Band-Limited Interpolation

Example

$$\begin{pmatrix} x(2) \\ x(3) \\ x(5) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{pmatrix}$$

Band-limited Interpolation
Minimum Norm Reconstruction

Minimize	$\|\|\mathbf{x}\|\|_{\mathcal{W}}^2$
Subject to	$\underset{\sim}{\mathbf{T}}\mathbf{x} = \mathbf{y}$

$$\|\|\mathbf{x}\|\|_{\mathcal{W}}^2 = \sum_{k \in \mathcal{K}} \frac{X_k^* X_k}{|P_k|^2}$$

$$\mathbf{X} = [X_0, X_1, \ldots, X_{M-1}]^T, \quad \mathbf{X} = \underset{\sim}{\mathbf{F}}\mathbf{x}$$

Band-limited Interpolation
Minimum Norm Reconstruction

$$||\mathbf{x}||_{\mathcal{W}}^2 = \sum_{k \in \mathcal{K}} \frac{X_k^* X_k}{|P_k|^2}$$

i) $|P_k|^2$ are spectral domain weights with support and shape similar to the spectrum of the signal to interpolate
ii) $\mathcal{K}$ indicates the region of spectral support of the signal
iii) $|P_k| \neq 0$ for $k \in \mathcal{K}$

Band-limited Interpolation

Minimum Norm Reconstruction

Define the following diagonal matrices

$$\Lambda_k = \begin{cases} |P_k|^2 & k \in \mathcal{K} \\ 0 & k \notin \mathcal{K} \end{cases}$$

$$\Lambda^{\dagger}{}_k = \begin{cases} |P_k|^{-2} & k \in \mathcal{K} \\ 0 & k \notin \mathcal{K} \end{cases}$$

$$\begin{aligned} ||\mathbf{x}||^2_{\mathcal{W}} &= \mathbf{x}^H \, \underset{\sim}{\mathbf{F}}^H \, \underset{\sim}{\mathbf{\Lambda}}^{\dagger} \, \underset{\sim}{\mathbf{F}} \, \mathbf{x} \\ &= \mathbf{x}^H \, \underset{\sim}{\mathbf{Q}}^{\dagger} \, \mathbf{x} \end{aligned}$$

Band-limited Interpolation

Minimum Norm Reconstruction

$$\begin{aligned} ||\mathbf{x}||^2_{\mathcal{W}} &= \mathbf{x}^H \underset{\sim}{\mathbf{F}}^H \underset{\sim}{\mathbf{\Lambda}}^\dagger \underset{\sim}{\mathbf{F}} \mathbf{x} \\ &= \mathbf{x}^H \underset{\sim}{\mathbf{Q}}^\dagger \mathbf{x} \end{aligned}$$

i) $\underset{\sim}{\mathbf{Q}}^\dagger = \underset{\sim}{\mathbf{F}}^H \underset{\sim}{\mathbf{\Lambda}}^\dagger \underset{\sim}{\mathbf{F}}$ is a circulant matrix

ii) $\underset{\sim}{\mathbf{Q}}^\dagger$ is a band-limiting operator, it annihilates any spectral component $k \notin \mathcal{K}$.

Band-limited Interpolation

Minimum Norm Reconstruction

Minimum norm inversion, minimize cost function J:

$$J = \lambda^T(\underset{\sim}{\mathbf{T}}\mathbf{x} - \mathbf{y}) + ||\mathbf{x}||^2_{\mathcal{W}}$$

$$\hat{\mathbf{x}} = \underset{\sim}{\mathbf{Q}}\underset{\sim}{\mathbf{T}}^T(\underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{Q}}\underset{\sim}{\mathbf{T}}^T)^{-1}\mathbf{y}\,.$$

i) In general we replace the inverse by $(\underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{Q}}\underset{\sim}{\mathbf{T}}^T)^\dagger$
ii) Assume a unit amplitude full-band operator , $\underset{\sim}{\mathbf{Q}} = \underset{\sim}{\mathbf{I}} \longrightarrow$

$$\hat{\mathbf{x}} = \underset{\sim}{\mathbf{T}}^T(\underset{\sim}{\mathbf{T}}\underset{\sim}{\mathbf{T}}^T)^{-1}\mathbf{y} = \underset{\sim}{\mathbf{T}}^T\mathbf{y}\,.$$

Band-limited interpolation
Minimum norm reconstruction - Example

$$\mathbf{y} = \underset{\sim}{\mathbf{T}}\mathbf{x}$$

$$\begin{pmatrix} x(2) \\ x(3) \\ x(5) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{pmatrix}$$

$$\hat{\mathbf{x}} = \underset{\sim}{\mathbf{T}}^T \mathbf{y} = \begin{pmatrix} 0 \\ x(2) \\ x(3) \\ 0 \\ x(5) \end{pmatrix}$$ Ooophs !!

Band-limited interpolation

Reconstruction of noisy data

Minimize cost function J:

$$J = ||\underset{\sim}{\mathbf{T}}\mathbf{x} - \mathbf{y}||^2 + \rho^2 ||\mathbf{x}||^2_{\mathcal{W}}$$

Band-limited interpolation

Reconstruction of noisy data

Or, solve the equivalent problem:

$$\begin{pmatrix} \underset{\sim}{\mathbf{T}} \\ \rho \underset{\sim}{\mathbf{W}} \end{pmatrix} \mathbf{x} \approx \begin{pmatrix} \mathbf{y} \\ \mathbf{0} \end{pmatrix} .$$

$$\underset{\sim}{\mathbf{W}} = \underset{\sim}{\mathbf{F}}^{H} \, \underset{\sim}{\mathbf{\Lambda}}^{1/2} \, \underset{\sim}{\mathbf{F}}$$

i) The augmented matrix of the problem is rank deficient

ii) Solve it by choosing, among all possible least-squares solutions, the one with minimum Euclidean norm

iii) SVD or CG

Band-limited interpolation

Reconstruction of noisy data - PCG

Change of variable: $\mathbf{x} = \mathbf{W}\mathbf{z}$

$$\begin{pmatrix} \underset{\sim}{\mathbf{T}}\,\underset{\sim}{\mathbf{W}}^{\dagger} \\ \rho \end{pmatrix} \mathbf{z} \approx \begin{pmatrix} \mathbf{y} \\ \mathbf{0} \end{pmatrix} .$$

i) The trade-off parameter can be set to $\rho = 0$ and use the number of iterations in the CG method play the role of regularization parameter (Hansen, 1998)

ii) Solve $\underset{\sim}{\mathbf{T}}\,\underset{\sim}{\mathbf{W}}^{\dagger}\mathbf{z} \approx \mathbf{y}$ and stop the algorithm when a target misfit is achieved

Band-limited interpolation

Reconstruction of noisy data - PCG

Weights can be estimated in an iterative manner using the Modified Peridogram

$$|P_k|^2 = \begin{cases} \sum_{l=-L}^{L} w_l |X_{k-l}|^2 \,, & k \in \mathcal{K} \\ 0\,, & k \notin \mathcal{K} \end{cases}$$

or, adopt a non-iterative strategy (Herrmann et al., 2000).

i) The weights $|P_k|^2$ required to interpolate spatial data at a temporal frequency f can be estimated from the already interpolated data at frequency $f - \Delta f$.

ii) Helps to reduce alias (mild alias!)

Band-limited interpolation - 2D case

Reconstruction of noisy data - PCG

$$\underset{\sim}{\mathbf{T}}\,\underset{\sim}{\mathbf{W}}^{\dagger}\,\mathbf{z} \approx \mathbf{y}$$

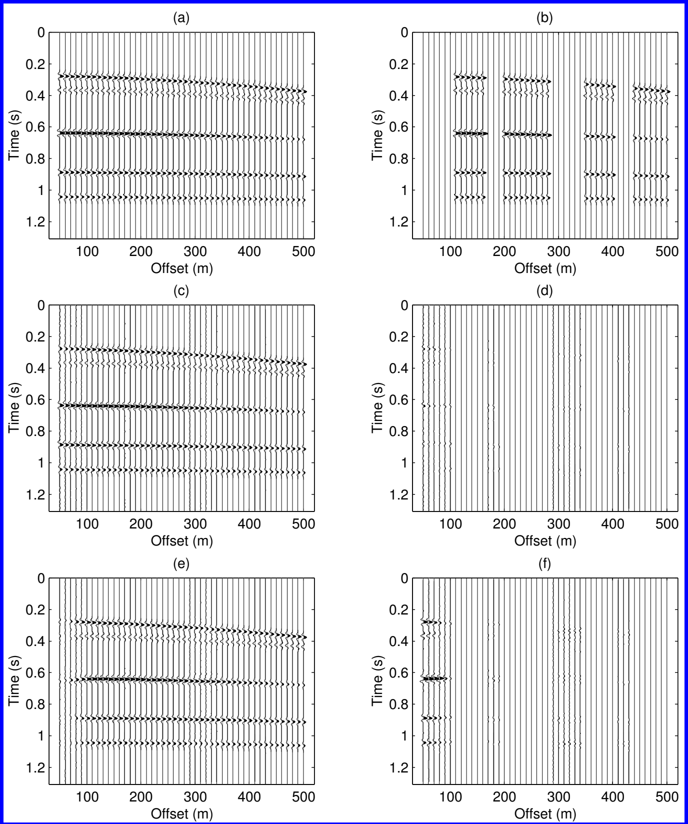
$$\underset{\sim}{\mathbf{W}} \;=\; (\underset{\sim}{\mathbf{F}}_u \otimes \underset{\sim}{\mathbf{F}}_v)^H\,\underset{\sim}{\mathbf{\Lambda}}^{1/2}\,(\underset{\sim}{\mathbf{F}}_u \otimes \underset{\sim}{\mathbf{F}}_v)\,.$$

i) The Kronecker tensor product is not needed

ii) Inner products required by PCG are computed in the flight

iii) Generalization to ND is straightforward

EXAMPLE 1 - MWNI vs MNI

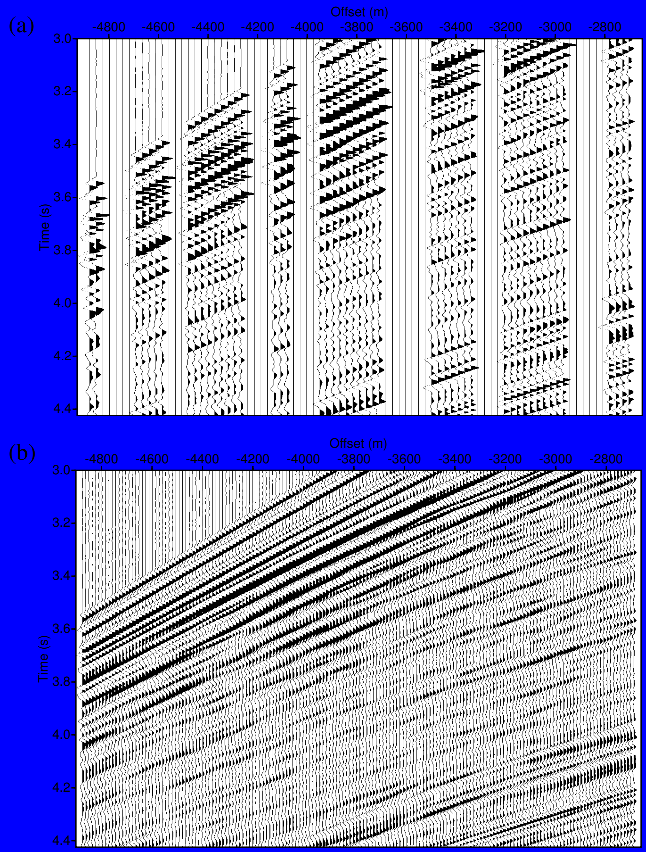


a) True, b) Decimated, c) MWNI reconstruction, d) MWNI error panel, e) MNI reconstruction, and f) MWNI error panel.

EXAMPLE 2 - Gulf of Mexico CSG

- MWNI is used to recover missing data (gaps) and to interpolate ($\Delta h_{new} = 1/2 \Delta h_{old}$)

EXAMPLE 2 - Gulf of Mexico CSG

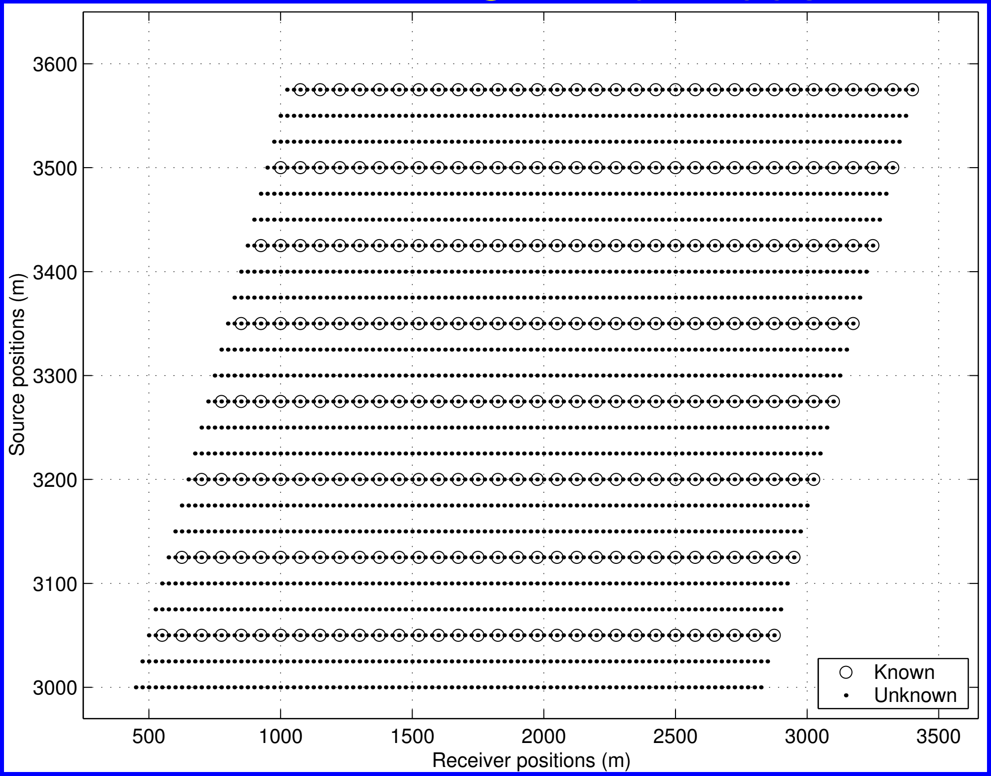


EXAMPLE 3 - Marmousi

1 - Data are decimated in both receicer/shot

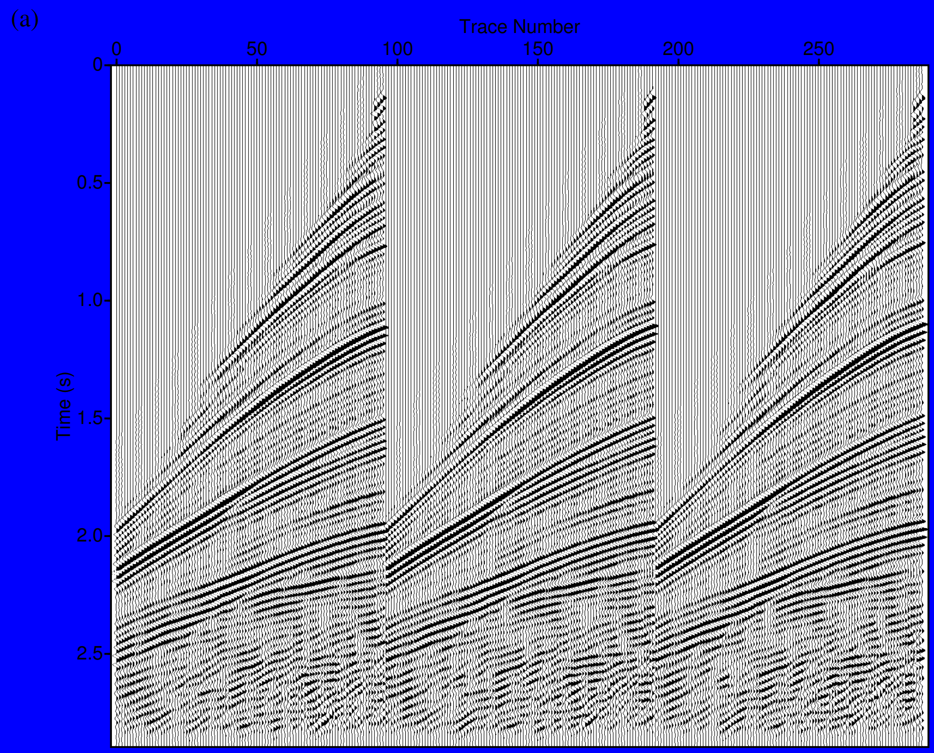
2 - MWNI is used to recover "decimated" positions

EXAMPLE 3 - Marmousi



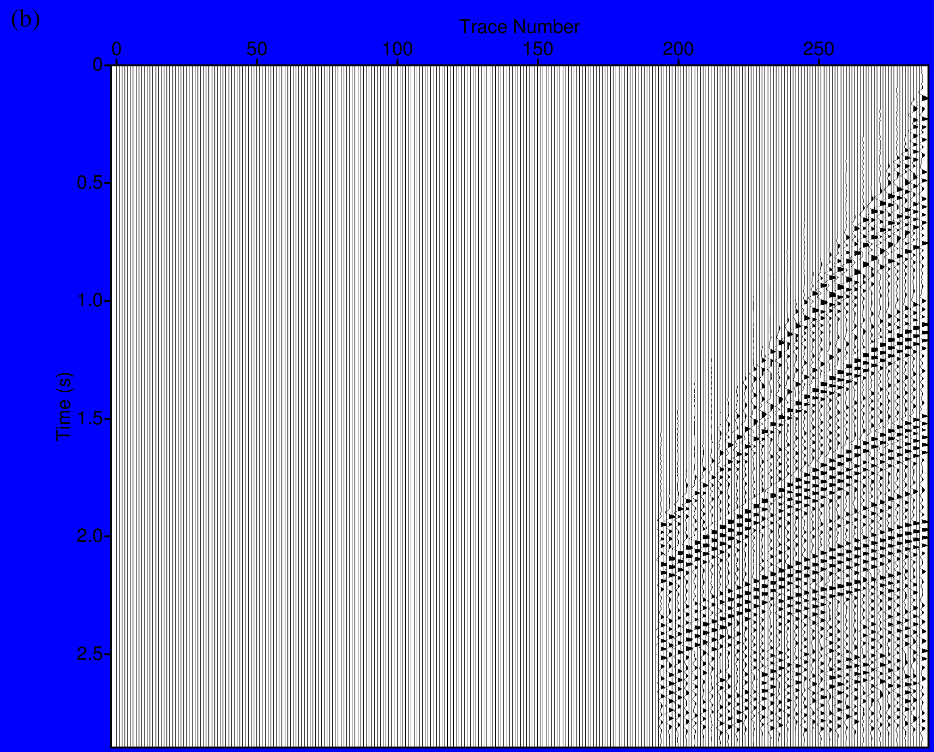
Data used in the numerical experiment

EXAMPLE 3 - Marmousi



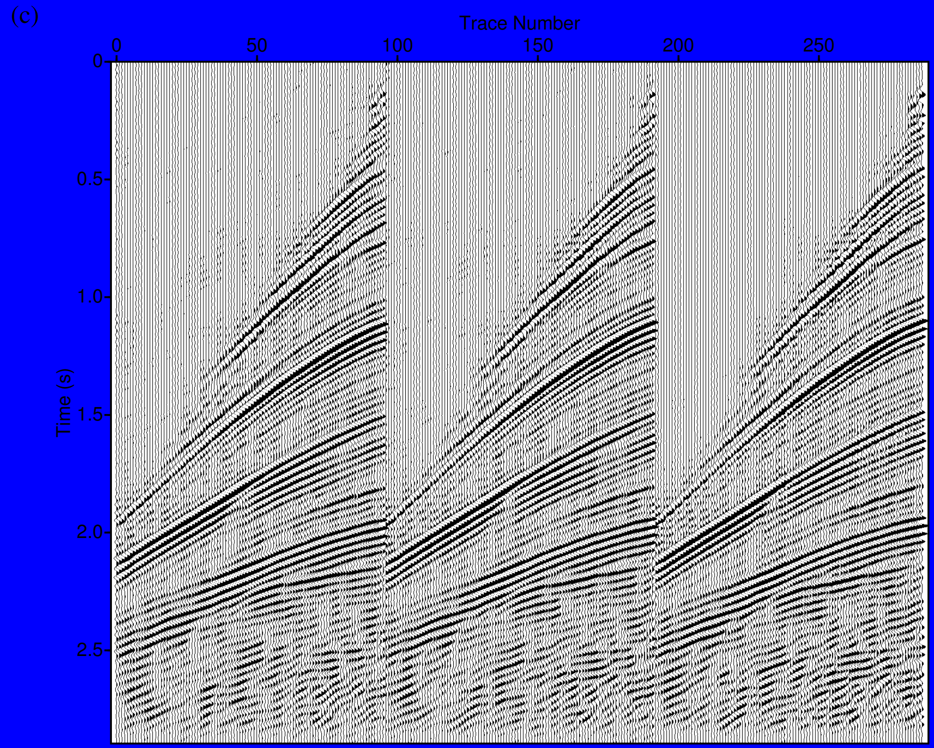
Shots at source positions $3075, 3100, 3125\,m$.

EXAMPLE 3 - Marmousi



Shots at source position $3075, 3100, 3125\,m$ after decimation.

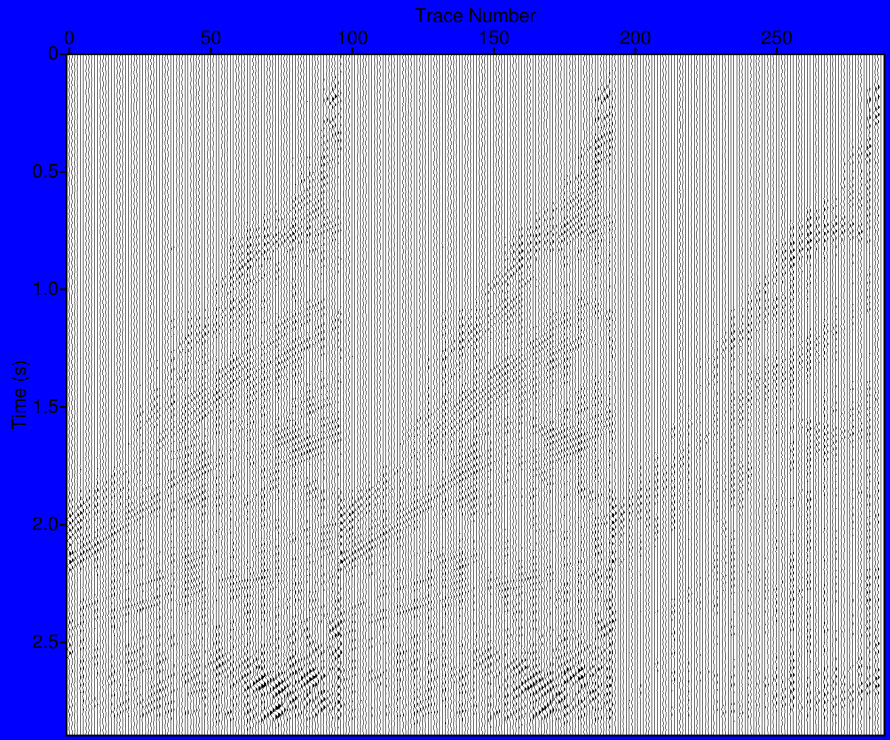
EXAMPLE 3 - Marmousi



MWNI Reconstuction, source position $3075, 3100, 3125\, m$.

EXAMPLE 3 - Marmousi

(d)

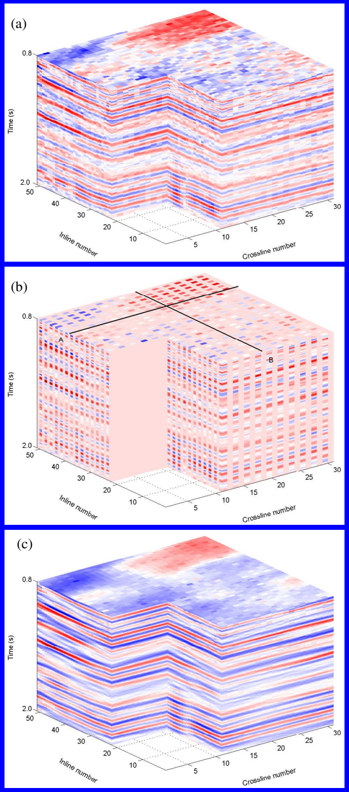


Error panel: Recostructed shots minus original shots.

EXAMPLE 4 - 3D WCB

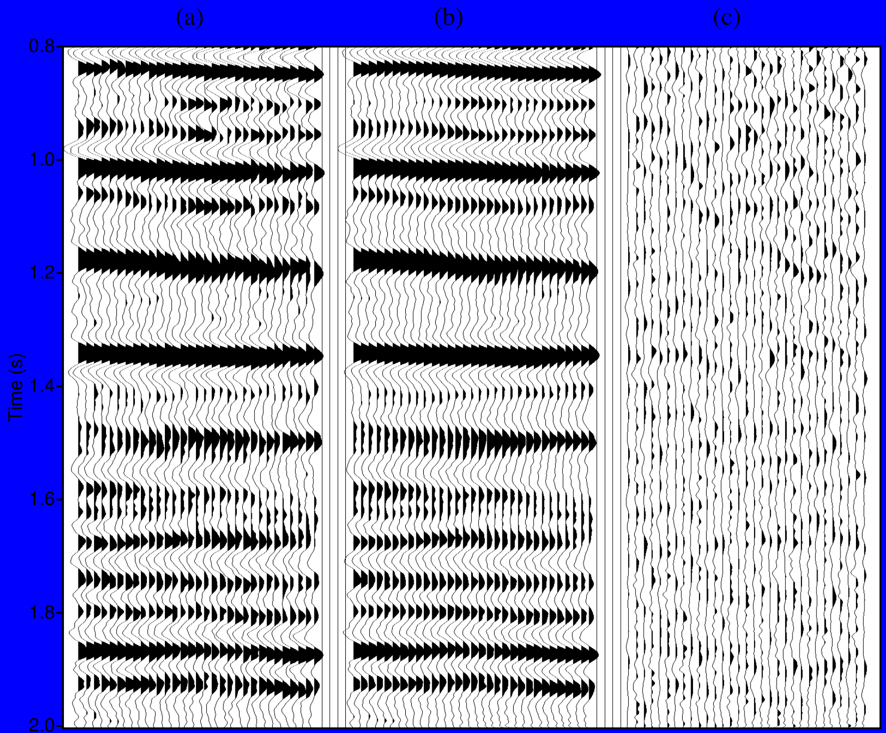
Post stack Interpolation and SNR Enhancement

EXAMPLE 4 - 3D WCB



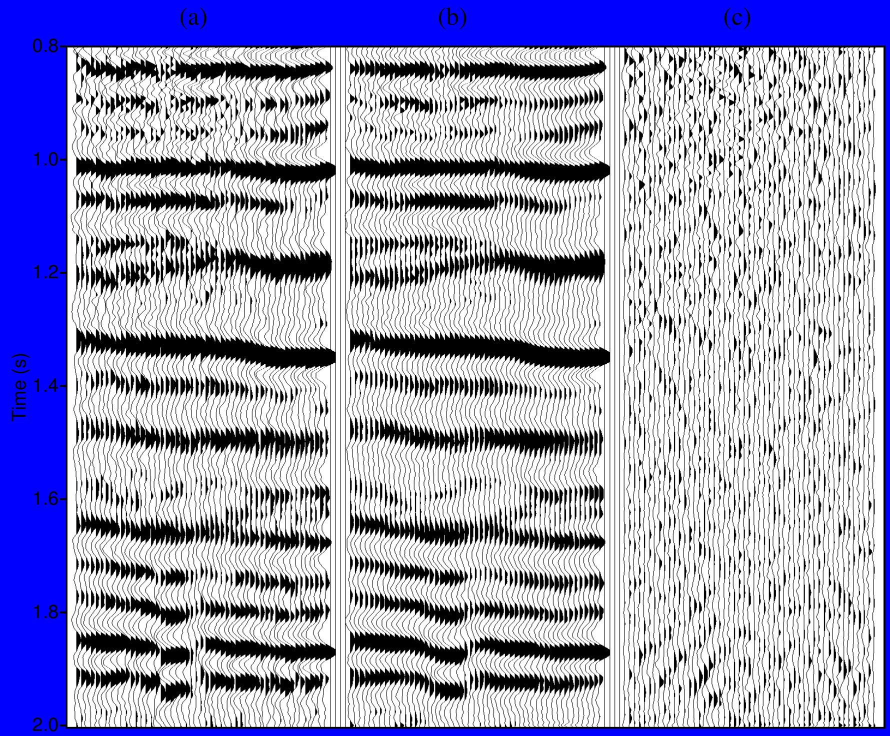
a) Original Post-stack cube. b) Decimated cube. c) reconstucted cube with MWNI.

EXAMPLE 4 - 3D WCB



a) Original. b) Reconstructed with MWNI. c) error.

EXAMPLE 4 - 3D WCB



a) Original. b) Reconstructed with MWNI. c) error.

EXAMPLE 5 - WCB Sparse 3D (Erskine)
Veritas

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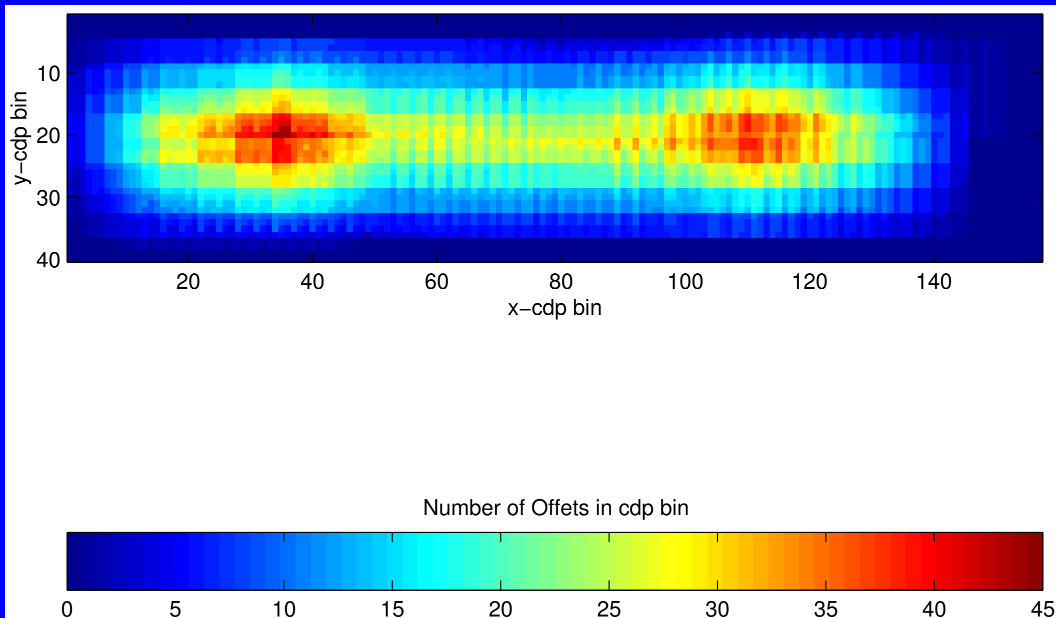
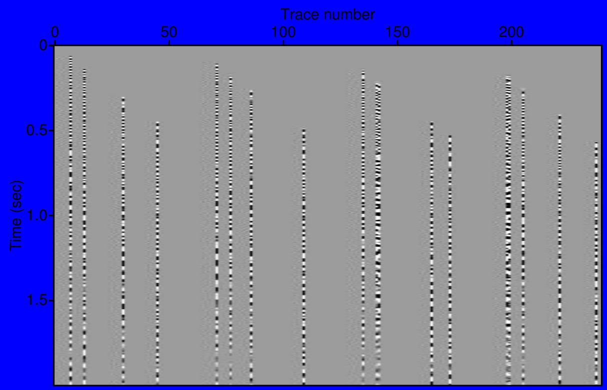


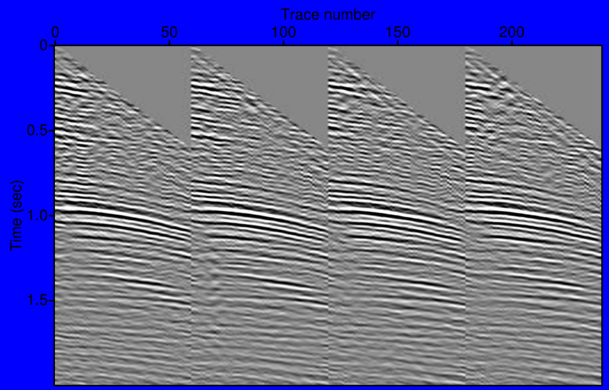
Interpolation to a regular x_m, y_m, h_x grid

is required before Common Azimuth AVP Wave Equation Migration

(Kuehl and Sacchi, SEG2001; Biondi et. al, EAGE2002)

EXAMPLE 5 - WCB Sparse 3D (Erskine)

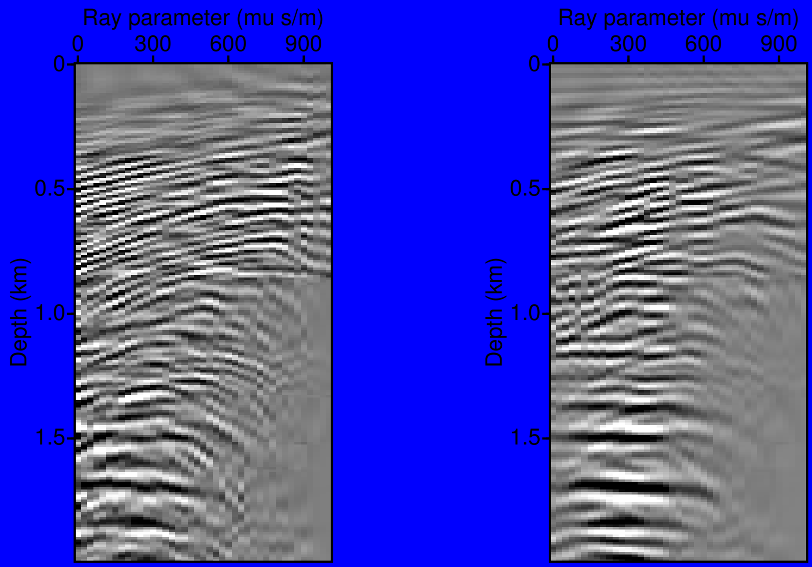




Inline 6, Crossline 15 to 18

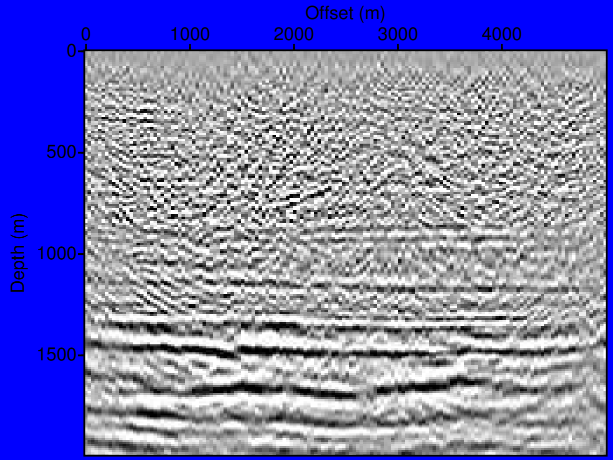
Left: Original. Right: After MWNI

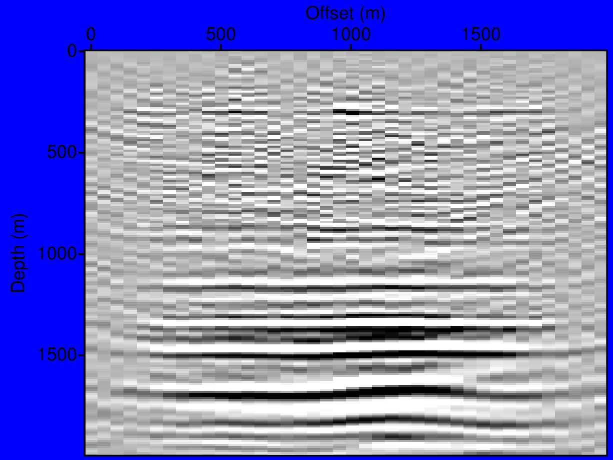
EXAMPLE 5 - WCB Sparse 3D (Erskine)



CIG (AVP) gathers. Left: No interpolation. Right: After MWNI

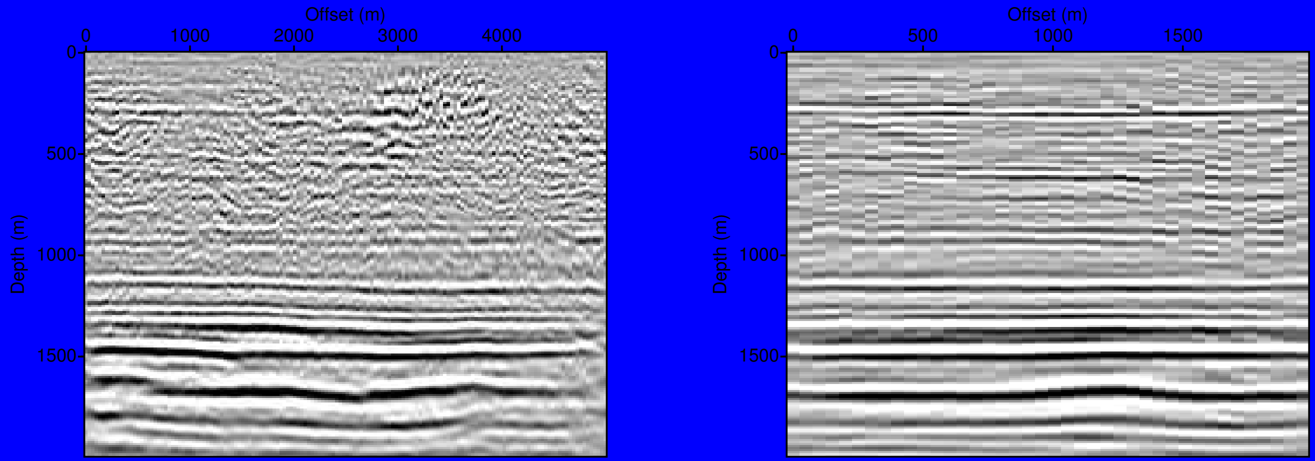
EXAMPLE 5 - WCB Sparse 3D (Erskine)





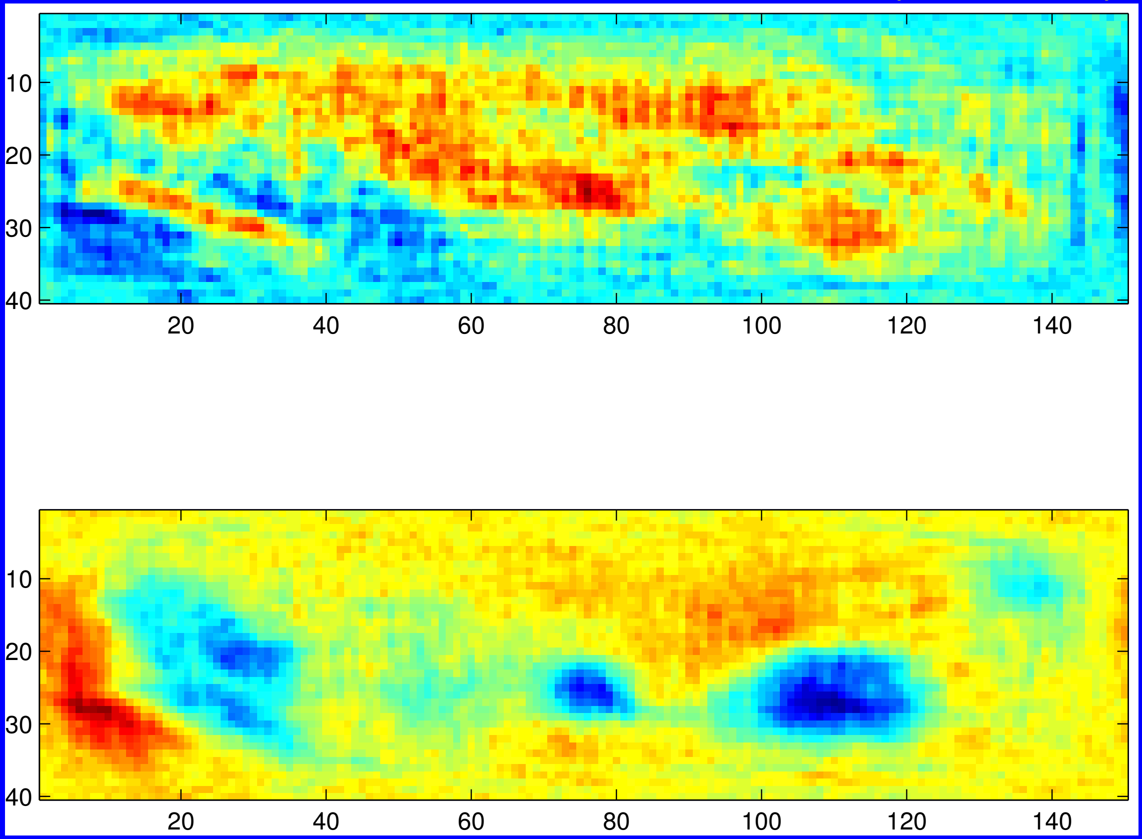
Migration [NO intepolation] Left: inline 36. Right: xline 71

EXAMPLE 5 - WCB Sparse 3D (Erskine)



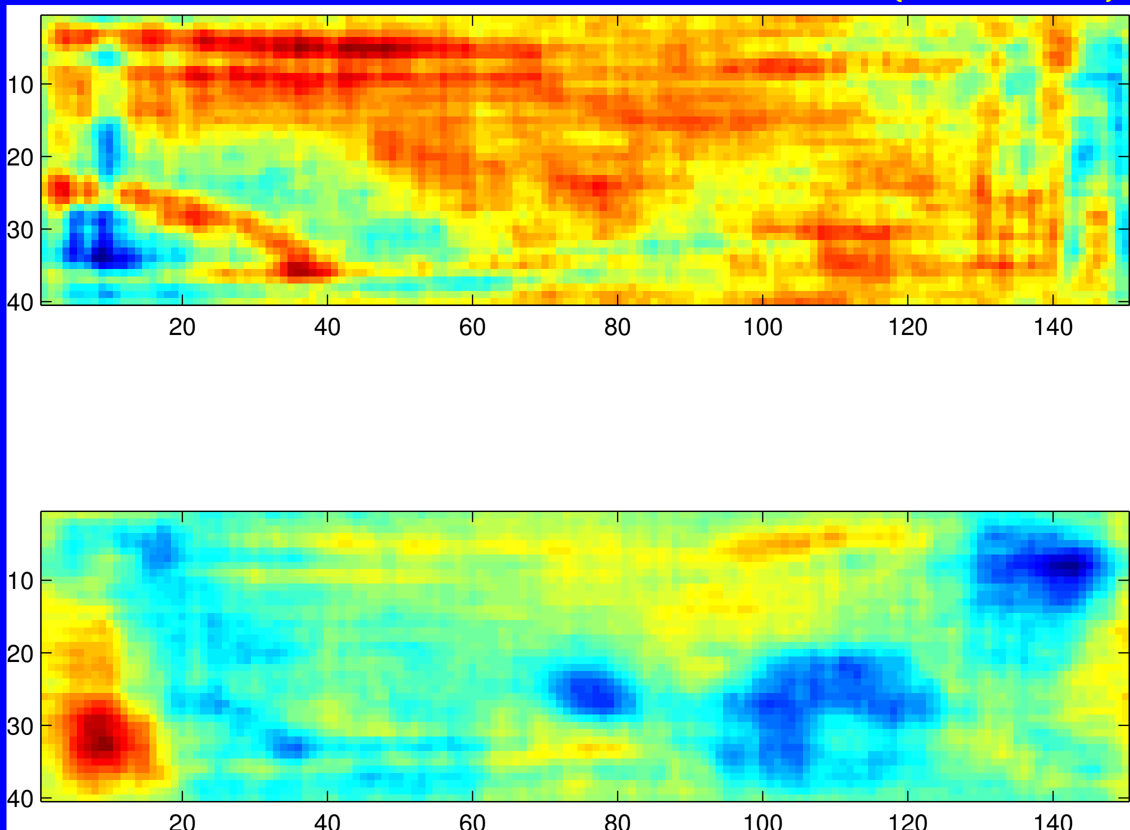
Migration after MWNI. Left: inline 36. Right: xline 71

EXAMPLE 5 - WCB Sparse 3D (Erskine)



Depth slices [NO interpolation] Top: $z = 1420\,m$. Bottom: $z = 1620\,m$

EXAMPLE 5 - WCB Sparse 3D (Erskine)



Depth slices after interpolation. Top: $z = 1420\,m$. Bottom: $z = 1620\,m$

EXAMPLE 5 - WCB Sparse 3D (Erskine)
All AVP Gathers

www-geo.phys.ualberta.ca/~sacchi/movies.html

Summary

i) MWNI: Efficient and simple (PCG and FFT)
ii) Better than MNI at the time of handling large gaps
iii) Simple to implement in any domain

Near future work:

i) Compare with AVA gathers obtained via LS migration
ii) Continue with amplitude fidelity studies (Kuehl and Sacchi, Geophysis 03)
iii) Alternatives (Radon, Azimuth Moveout, etc)

References [Interpolation]

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